4.11 Example of fault calculation for three phase and LG faults in power system network

A single line diagram of a power system is shown in Fig. 4.67 and the system data is as follows:-

- **Generators G_1 and G_2** : $\bar{X}_1 = \bar{X}_2 = 0.2\ pu, \bar{X}_0 = 0.05\ pu$
- **Transformers T_1 and T_2** : $\bar{X}_1 = \bar{X}_2 = \bar{X}_0 = \bar{X}_\ell = 0.05\ pu$
- **Transmission Lines $L_1,\ L_2$ and L_3** : $\bar{X}_1 = \bar{X}_2 = 0.1\ pu, \bar{X}_0 = 0.3\ pu$

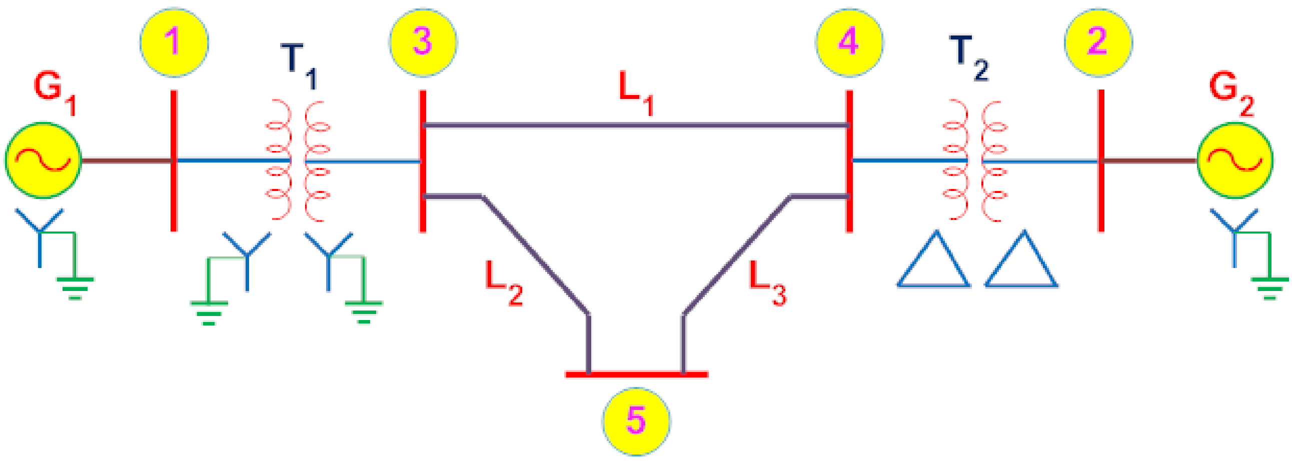

Figure 4.67: Single line diagram of the power System of the example

Prefault voltage for all buses is taken as $\bar{V}_i(0) = 1.0\angle 0^0\ pu\ \forall\ i = 1, 2, 3$.

We wish to carry out the complete short-circuit analysis of the system for:

(a) three phase bolted fault at bus 5

(b) LG fault with $\bar{Z}_f = 0.1$ pu at bus 5

(c) LL fault with $\bar{Z}_f = 0.1$ pu at bus 5

(d) LLG fault with $\bar{Z}_f = 0.0$ pu at bus 5

Solution:

(a) Three phase fault at bus 5

For the three phase bolted fault, only positive sequence network and the positive sequence bus impedance matrix $[\bar{\mathbf{Z}}^{(1)}_{\mathbf{Bus}}]$ is required. The positive sequence network for the power system of Fig. 4.67 is shown in Fig. 4.68. In this diagram all the elements have been replaced by their per unit positive sequence impedances.

The $[\bar{\mathbf{Z}}^{(1)}_{\mathbf{Bus}}]$ matrix for the network of the Fig. 4.68 is given below:

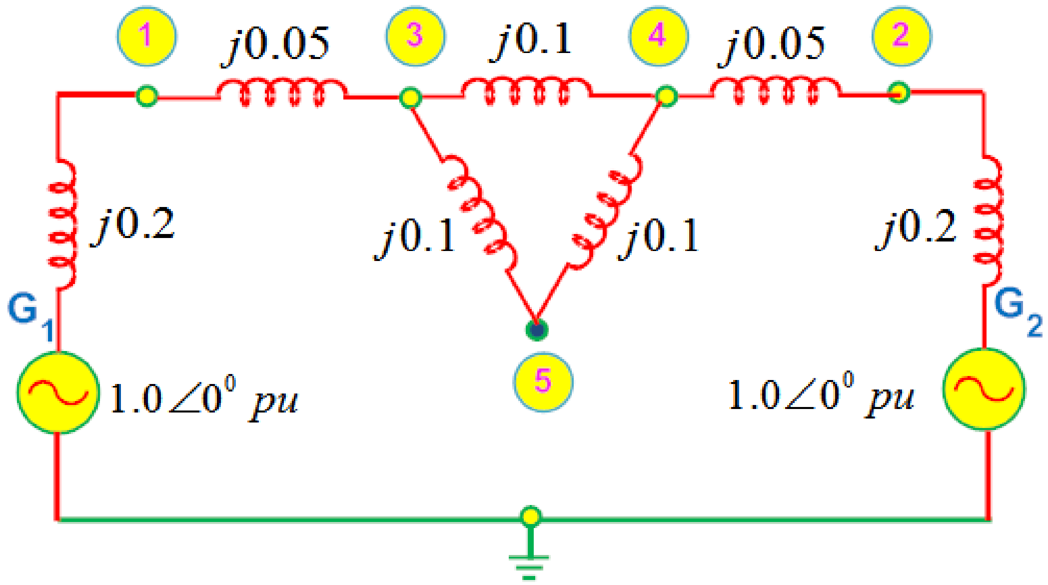

Figure 4.68: Positive sequence equivalent network of Fig. 4.67

$$[\bar{\mathbf{Z}}^{(1)}_{\mathbf{Bus}}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} j0.1294 & j0.0706 & j0.1118 & j0.0882 & j0.10 \\ j0.0706 & j0.1294 & j0.0882 & j0.1118 & j0.10 \\ j0.1118 & j0.0882 & j0.1397 & j0.1103 & j0.1250 \\ j0.0882 & j0.1118 & j0.1103 & j0.1397 & j0.1250 \\ j0.10 & j0.10 & j0.1250 & j0.1250 & j0.1750 \end{bmatrix} \end{array} pu$$

The sequence component of three phase fault current at **bus 5** are given as, from equation (4.73):

$$\left[\bar{\mathbf{I}}^{(012)}_{5}(\mathbf{F})\right] = \begin{bmatrix} 0 \\ \frac{1}{\bar{Z}^1_{55}} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{j0.1750} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -j5.7143 \\ 0 \end{bmatrix} pu$$

The phase components of the fault current are calculated using equation (4.122):

$$[\bar{\mathbf{I}}^{(abc)}_{5}(\mathbf{F})] = [\bar{\mathbf{A}}][\bar{\mathbf{I}}^{(012)}_{5}(\mathbf{F})]$$

$$\left[\bar{\mathbf{I}}^{(abc)}_{\mathbf{fault}}\right] = \left[\bar{\mathbf{I}}^{(abc)}_{5}(\mathbf{F})\right] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a^2} & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a^2} \end{bmatrix} \begin{bmatrix} 0 \\ -j5.7143 \\ 0 \end{bmatrix} = \begin{bmatrix} 5.7143\angle -90^0 \\ 5.7143\angle 150^0 \\ 5.7143\angle 30^0 \end{bmatrix} pu$$

Bus voltages during fault

Bus 1:

$$\begin{aligned}\bar{V}_1^{(1)}(F) &= \bar{V}_1^{(1)}(0) - \bar{Z}_{15}^{(1)}\bar{I}_5^{(1)}(F) \\ &= 1.0 - j0.10 * (-j5.7143) \\ &= 0.42857\angle 0^0 pu\end{aligned}$$

Since this is a balanced fault, $\bar{V}_1^{(a)}(F) = \bar{V}_1^{(1)}(F)$

$$\left[\mathbf{V_1^{(abc)}(F)}\right] = \begin{bmatrix} 0.42857\angle 0^0 \\ 0.42857\angle -120^0 \\ 0.42857\angle 120^0 \end{bmatrix} pu$$

Bus 2:

$$\begin{aligned}\bar{V}_2^{(1)}(F) &= \bar{V}_2^{(1)}(0) - \bar{Z}_{25}^{(1)}\bar{I}_5^{(1)}(F) \\ &= 1.0 - j0.10 * (-j5.7143) \\ &= 0.42857\angle 0^0 pu\end{aligned}$$

Since this is a balanced fault $\bar{V}_2^{(a)}(F) = \bar{V}_2^{(1)}(F)$

$$\left[\mathbf{V_2^{(abc)}(F)}\right] = \begin{bmatrix} 0.42857\angle 0^0 \\ 0.42857\angle -120^0 \\ 0.42857\angle 120^0 \end{bmatrix} pu$$

Bus 3:

$$\begin{aligned}\bar{V}_3^{(1)}(F) &= \bar{V}_3^{(1)}(0) - \bar{Z}_{35}^{(1)}\bar{I}_5^{(1)}(F) \\ &= 1.0 - j0.125 * (-j5.7143) \\ &= 0.28571\angle 0^0 pu\end{aligned}$$

Since this is a balanced fault $\bar{V}_3^{(a)}(F) = \bar{V}_3^{(1)}(F)$

$$\left[\mathbf{V_3^{(abc)}(F)}\right] = \begin{bmatrix} 0.28571\angle 0^0 \\ 0.28571\angle -120^0 \\ 0.28571\angle 120^0 \end{bmatrix} pu$$

Bus 4:

$$\begin{aligned}\bar{V}_4^{(1)}(F) &= \bar{V}_4^{(1)}(0) - \bar{Z}_{45}^{(1)}\bar{I}_5^{(1)}(F)\\ &= 1.0 - j0.125 * (-j5.7143)\\ &= 0.28571\angle 0^0 pu\end{aligned}$$

Since this is a balanced fault $\bar{V}_4^{(a)}(F) = \bar{V}_4^{(1)}(F)$

$$\left[\mathbf{V_4^{(abc)}(F)}\right] = \begin{bmatrix} 0.28571\angle 0^0 \\ 0.28571\angle -120^0 \\ 0.28571\angle 120^0 \end{bmatrix} pu$$

The bus voltage of **bus 5** under faulted condition is $\mathbf{\bar{V}_5^{(abc)}(F)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} pu$ because the fault impedance is zero.

Line Currents during fault

For line L_1 from **bus 3** to **bus 4** the positive sequence component for line current $\left(\bar{I}_{34}^{(1)}(F)\right)$ is calculated as:

$$\bar{I}_{34}^{(1)}(F) = \frac{\bar{V}_3^{(1)}(F) - \bar{V}_4^{(1)}(F)}{\bar{Z}_{34}^{(1)}} = \frac{0.28571 - 0.28571}{j0.1} = 0$$

Hence, the phase components of line current are

$$\mathbf{\bar{I}_{34}^{(abc)}(F)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} pu$$

For line L_2 from **bus 3** to **bus 5** the positive sequence component for line current $\left(\bar{I}_{35}^{(1)}(F)\right)$ is calculated as:

$$\bar{I}_{35}^{(1)}(F) = \frac{\bar{V}_3^{(1)}(F) - \bar{V}_5^{(1)}(F)}{\bar{Z}_{35}^{(1)}} = \frac{0.28571 - 0.0}{j0.1} = 2.8571\angle -90^0 pu$$

Hence, the phase components of line current are

$$\left[\mathbf{I_{35}^{(abc)}(F)}\right] = \begin{bmatrix} 2.8571\angle -90^0 \\ 2.8571\angle 150^0 \\ 2.8571\angle 30^0 \end{bmatrix} pu$$

For line L_3 from **bus 4** to **bus 5** the positive sequence component for line current $\left(\bar{I}_{45}^{(1)}(F)\right)$ is calculated as:

$$\bar{I}_{45}^{(1)}(F) = \frac{\bar{V}_4^{(1)}(F) - \bar{V}_5^{(1)}(F)}{\bar{Z}_{45}^{(1)}} = \frac{0.28571 - 0.0}{j0.1} = 2.8571\angle -90^0 pu$$

Hence, the phase components of line current are

$$\left[\mathbf{I_{45}^{(abc)}(F)}\right] = \begin{bmatrix} 2.8571\angle -90^0 \\ 2.8571\angle 150^0 \\ 2.8571\angle 30^0 \end{bmatrix} pu$$

Transformer Currents during fault

For transformer T_1 between **bus 1** and **bus 3** the positive sequence component fault current $\left(\bar{I}_{13}^{(1)}(F)\right)$ is calculated as:

$$\bar{I}_{13}^{(1)}(F) = \frac{\bar{V}_1^{(1)}(F) - \bar{V}_3^{(1)}(F)}{\bar{z}_{T_1}^{(1)}} = \frac{0.42857 - 0.28571}{j0.05} = 2.8571\angle -90^0 pu$$

The phase components of the transformer T_1 current are:

$$\left[\mathbf{\bar{I}_{31}^{(abc)}(F)}\right] = \begin{bmatrix} 2.8571\angle -90^0 \\ 2.8571\angle 150^0 \\ 2.8571\angle 30^0 \end{bmatrix} pu$$

For transformer T_2 between **bus 2** and **bus 4** the positive sequence component fault current $\left(\bar{I}_{24}^{(1)}(F)\right)$ is calculated as:

$$\bar{I}_{24}^{(1)}(F) = \frac{\bar{V}_2^{(1)}(F) - \bar{V}_4^{(1)}(F)}{\bar{z}_{T_2}^{(1)}} = \frac{0.42857 - 0.28571}{j0.05} = 2.8571\angle -90^0 pu$$

The phase components of the transformer T_2 current are:

$$\left[\mathbf{\bar{I}_{24}^{(abc)}(F)}\right] = \begin{bmatrix} 2.8571\angle -90^0 \\ 2.8571\angle 150^0 \\ 2.8571\angle 30^0 \end{bmatrix} pu$$

Generator Currents during fault

For generator G_1 connected at **bus 1** the positive sequence component fault current $\left(\bar{I}_{G_1}^{(1)}(F)\right)$ is calculated as:

$$\bar{I}_{G_1}^{(1)}(F) = \frac{\bar{E}_a - \bar{V}_1^{(1)}(F)}{\bar{z}_{G_2}^{(1)}} = \frac{1.0 - 0.42857}{j0.2} = 2.8571\angle -90^0 pu$$

The phase components of the generator G_1 current are:

$$\left[\bar{\mathbf{I}}_{\mathbf{G_1}}^{(\mathbf{abc})}(\mathbf{F})\right] = \begin{bmatrix} 2.8571\angle -90^0 \\ 2.8571\angle 150^0 \\ 2.8571\angle 30^0 \end{bmatrix} pu$$

For Generator G_2 connected at **bus 2** the positive sequence component fault current $\left(\bar{I}_{G_2}^{(1)}(F)\right)$ is calculated as:

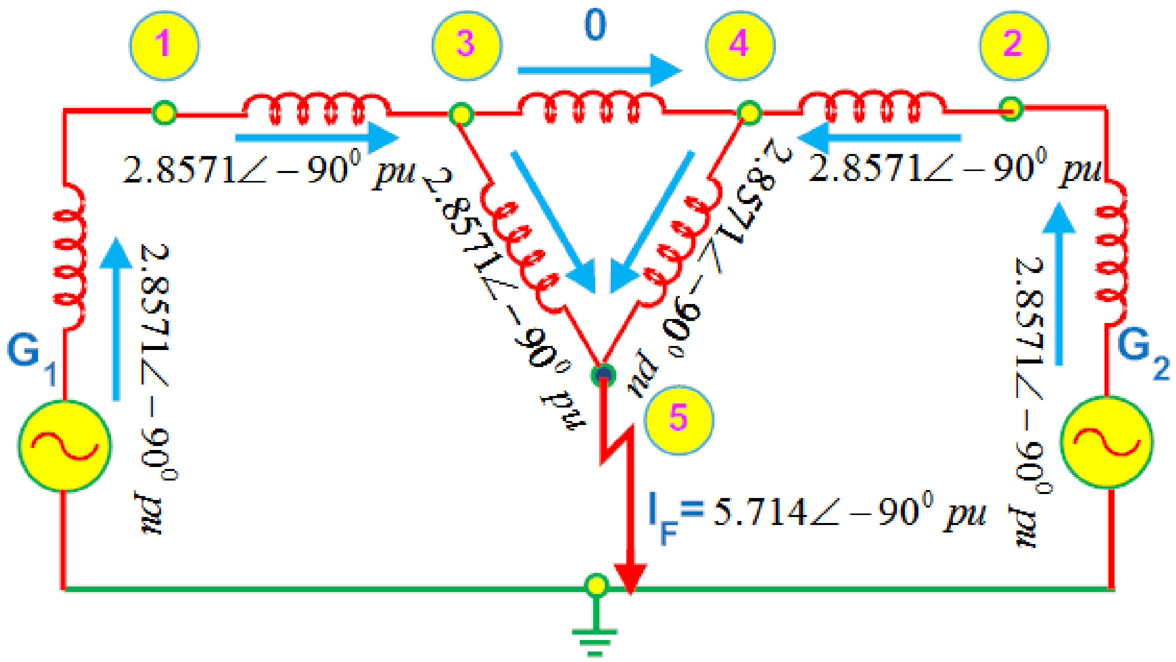

Figure 4.69: Flow of fault current in the network

$$\bar{I}_{G_2}^{(1)}(F) = \frac{\bar{E}_a - \bar{V}_2^{(1)}(F)}{\bar{z}_{G_2}^{(1)}} = \frac{1.0 - 0.42857}{j0.2} = 2.8571\angle -90^0 pu$$

The phase components of the generator G_2 current can be calculated as:

$$\left[\bar{\mathbf{I}}_{\mathbf{G_2}}^{(\mathbf{abc})}(\mathbf{F})\right] = \begin{bmatrix} 2.8571\angle -90^0 \\ 2.8571\angle 150^0 \\ 2.8571\angle 30^0 \end{bmatrix} pu$$

The flow of fault current in the system is shown in the single line diagram of Fig. 4.69.

(b) Single line to ground fault at bus 5

In this case all the sequence networks are required. The positive sequence network is same as the one shown in the Fig. 4.68 and $[\bar{\mathbf{Z}}^{(1)}_{\mathbf{Bus}}]$ is identical to the matrix used in three phase fault analysis.

The negative sequence equivalent network for this network is as shown in Fig. 4.70. The network

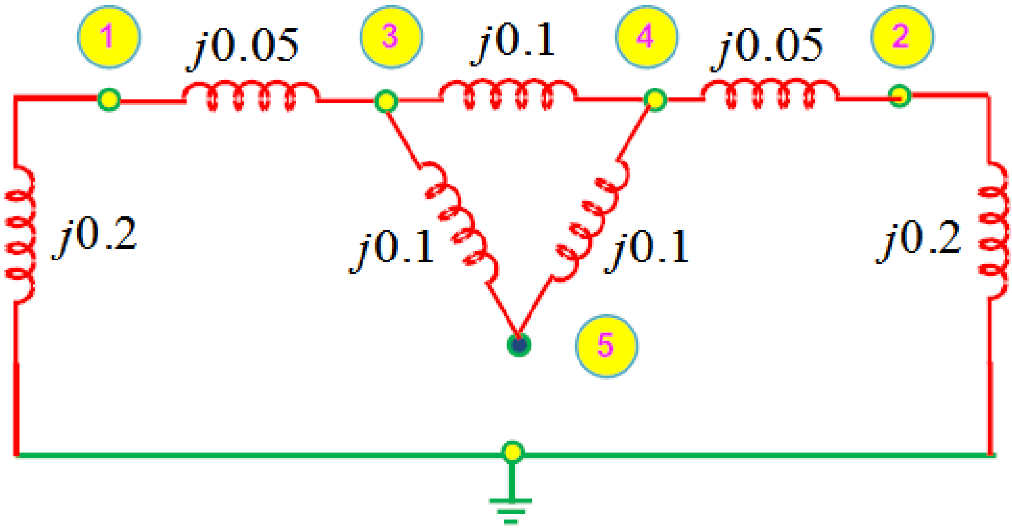

Figure 4.70: Negative sequence equivalent network

is identical to the network of Fig. 4.68 except for the voltage sources. Hence, $[\bar{\mathbf{Z}}^{(2)}_{\mathbf{Bus}}] = [\bar{\mathbf{Z}}^{(1)}_{\mathbf{Bus}}]$.

The zero sequence equivalent network is drawn next considering the transformer connections and grounding as well as generator grounding. The equivalent zero sequence networks is shown in Fig. 4.71.

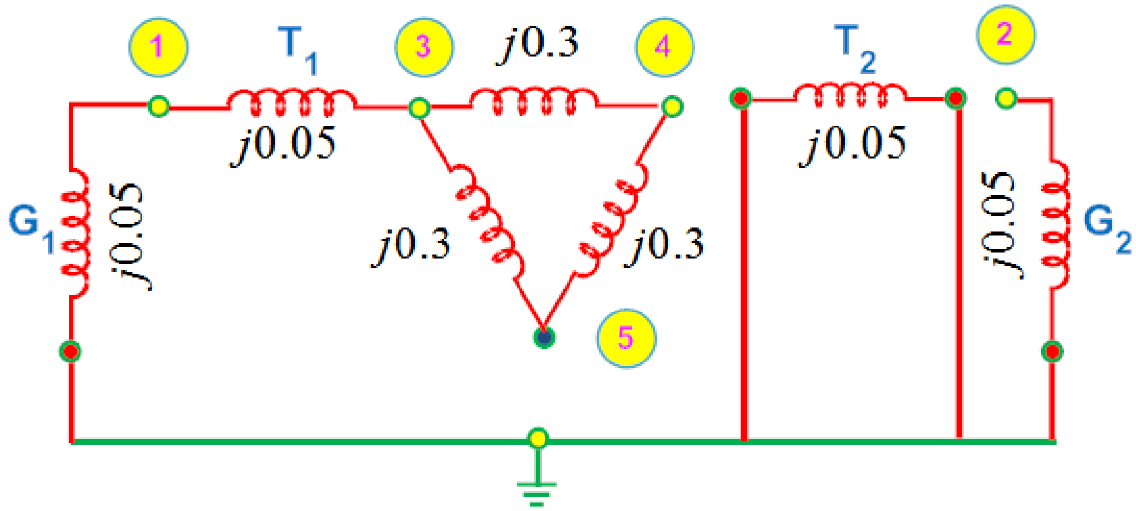

Figure 4.71: Zero sequence equivalent network

An explanation of the equivalent circuit will be in order. Generators G_1 *and* G_2 have their neutrals grounded, so their zero sequence impedances are connected to the reference. Transformer T_1 has both the windings connected in star, with both neutrals solidly grounded. As a result, the zero sequence impedance of the transformer is directly connected between **buses 1 and 2**.

Transformer T_2 has both the winding connected in delta, hence, no connection exists between the primary and secondary sides for zero sequence currents to flow. To represent circulating zero sequence currents in the delta connected transformer winding, it is represented as a short circuited winding.

$[\bar{\mathbf{Z}}^{(0)}_{\mathbf{Bus}}]$, the zero sequence bus impedance matrix is then calculated using the step-by-step $\mathbf{Z_{Bus}}$ building algorithm. The zero sequence bus impedance matrix is given as:

$$[\bar{\mathbf{Z}}^{(0)}_{\mathbf{Bus}}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ j0.05 & 0.0 & j0.05 & j0.05 & j0.05 \\ 0.0 & j0.05 & 0.0 & 0.0 & 0.0 \\ j0.05 & 0.0 & j0.10 & j0.10 & j0.10 \\ j0.05 & 0.0 & j0.10 & j0.30 & j0.20 \\ j0.05 & 0.0 & j0.10 & j0.20 & j0.30 \end{bmatrix} pu$$

Fault current:

The sequence component of the fault current at **bus 5** are given as, from equation (4.121):

$$\bar{I}_5^{(0)}(F) = \bar{I}_5^{(1)}(F) = \bar{I}_5^{(2)}(F) = \frac{\bar{V}_k(0)}{\bar{Z}_{55}^{(1)} + \bar{Z}_{55}^{(2)} + \bar{Z}_{55}^{(0)}} = \frac{1.0}{j0.175 + j0.175 + j0.30} = -j1.538\, pu$$

$$\left[\bar{\mathbf{I}}^{(\mathbf{abc})}_{\mathbf{fault}}\right] = \left[\bar{\mathbf{I}}^{(\mathbf{abc})}_{\mathbf{5}}(\mathbf{F})\right] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j1.538 \\ -j1.538 \\ -j1.538 \end{bmatrix}$$

$$\left[\bar{\mathbf{I}}^{(\mathbf{abc})}_{\mathbf{fault}}\right] = \begin{bmatrix} 4.6154\angle -90^0 \\ 0 \\ 0 \end{bmatrix} pu$$

Bus voltages:

The bus voltage in sequence components, during fault, are calculated using equation (4.127) written in compact form as:

$$\begin{bmatrix} \bar{V}_i^{(0)}(F) \\ \bar{V}_i^{(1)}(F) \\ \bar{V}_i^{(2)}(F) \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{V}_i \\ 0 \end{bmatrix} - \begin{bmatrix} \bar{Z}_{ik}^{(0)} & 0 & 0 \\ 0 & \bar{Z}_{ik}^{(1)} & 0 \\ 0 & 0 & \bar{Z}_{ik}^{(2)} \end{bmatrix} \begin{bmatrix} \bar{I}_k^{(0)}(F) \\ \bar{I}_k^{(1)}(F) \\ \bar{I}_k^{(2)}(F) \end{bmatrix} \qquad (4.131)$$

where, **k** represents the faulted bus number and $\bar{Z}_{ik}^{(0)}, \bar{Z}_{ik}^{(1)}$ and $\bar{Z}_{ik}^{(2)}$ are the elements of the respective sequence bus impedance matrices.

$\bar{I}_k^{(0)}, \bar{I}_k^{(1)}$ and $\bar{I}_k^{(2)}$ represent the sequence components of fault current at $\mathbf{k^{th}}$ bus.

$\bar{V}_i(0)$ is the pre fault bus voltage of $\mathbf{i^{th}}$ bus.

Bus 1: The sequence voltages are:

$$\begin{bmatrix} \bar{V}_1^{(0)}(F) \\ \bar{V}_1^{(1)}(F) \\ \bar{V}_1^{(2)}(F) \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.05 & 0 & 0 \\ 0 & j0.10 & 0 \\ 0 & 0 & j0.10 \end{bmatrix} \begin{bmatrix} -j1.538 \\ -j1.538 \\ -j1.538 \end{bmatrix}$$

Or,

$$\left[\mathbf{\bar{V}_1^{(012)}(F)}\right] = \begin{bmatrix} -0.0769 \\ 0.8462 \\ -0.1538 \end{bmatrix} pu$$

The bus voltage in the phase form is calculated using equation (4.128).

$$\left[\mathbf{\bar{V}_1^{(abc)}(F)}\right] = \begin{bmatrix} 0.6154\angle 0^0 \\ 0.9638\angle -116.04^0 \\ 0.9638\angle -116.04^0 \end{bmatrix} pu$$

Bus 2: The sequence voltages are:

$$\begin{bmatrix} \bar{V}_2^{(0)}(F) \\ \bar{V}_2^{(1)}(F) \\ \bar{V}_2^{(2)}(F) \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0.0 & 0 & 0 \\ 0 & j0.10 & 0 \\ 0 & 0 & j0.10 \end{bmatrix} \begin{bmatrix} -j1.538 \\ -j1.538 \\ -j1.538 \end{bmatrix}$$

Or,

$$\left[\mathbf{\bar{V}_2^{(012)}(F)}\right] = \begin{bmatrix} 0.0 \\ 0.8462 \\ -0.1538 \end{bmatrix} pu$$

The bus voltage in the phase form is calculated using equation (4.128).

$$\left[\mathbf{\bar{V}_2^{(abc)}(F)}\right] = \begin{bmatrix} 0.6923\angle 0^0 \\ 0.9326\angle -111.79^0 \\ 0.9326\angle -111.79^0 \end{bmatrix} pu$$

Bus 3: The sequence voltages are:

$$\begin{bmatrix} \bar{V}_3^{(0)}(F) \\ \bar{V}_3^{(1)}(F) \\ \bar{V}_3^{(2)}(F) \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.10 & 0 & 0 \\ 0 & j0.125 & 0 \\ 0 & 0 & j0.125 \end{bmatrix} \begin{bmatrix} -j1.538 \\ -j1.538 \\ -j1.538 \end{bmatrix}$$

Or,

$$\left[\mathbf{\bar{V}_3^{(012)}(F)}\right] = \begin{bmatrix} -0.1538 \\ 0.8077 \\ -0.1923 \end{bmatrix} pu$$

The bus voltage in the phase form is calculated using equation (4.128).

$$\left[\bar{\mathbf{V}}_{\mathbf{3}}^{(\mathbf{abc})}(\mathbf{F})\right] = \begin{bmatrix} 0.4615\angle 0^0 \\ 0.9813\angle -118.05^0 \\ 0.9813\angle -118.05^0 \end{bmatrix} pu$$

Bus 4: The sequence voltages are:

$$\begin{bmatrix} \bar{V}_4^{(0)}(F) \\ \bar{V}_4^{(1)}(F) \\ \bar{V}_4^{(2)}(F) \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.20 & 0 & 0 \\ 0 & j0.125 & 0 \\ 0 & 0 & j0.125 \end{bmatrix} \begin{bmatrix} -j1.538 \\ -j1.538 \\ -j1.538 \end{bmatrix}$$

Or,

$$\left[\bar{\mathbf{V}}_{\mathbf{4}}^{(\mathbf{012})}(\mathbf{F})\right] = \begin{bmatrix} -0.3076 \\ 0.8077 \\ -0.1923 \end{bmatrix} pu$$

The bus voltage in the phase form is calculated using equation (4.128).

$$\left[\bar{\mathbf{V}}_{\mathbf{4}}^{(\mathbf{abc})}(\mathbf{F})\right] = \begin{bmatrix} 0.3077\angle 0^0 \\ 1.0624\angle -125.40^0 \\ 1.0624\angle -125.40^0 \end{bmatrix} pu$$

Bus 5: The sequence voltages are:

$$\begin{bmatrix} \bar{V}_5^{(0)}(F) \\ \bar{V}_5^{(1)}(F) \\ \bar{V}_5^{(2)}(F) \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.30 & 0 & 0 \\ 0 & j0.175 & 0 \\ 0 & 0 & j0.175 \end{bmatrix} \begin{bmatrix} -j1.538 \\ -j1.538 \\ -j1.538 \end{bmatrix}$$

Or,

$$\left[\bar{\mathbf{V}}_{\mathbf{5}}^{(\mathbf{012})}(\mathbf{F})\right] = \begin{bmatrix} -0.4615 \\ 0.7308 \\ -0.2692 \end{bmatrix} pu$$

The bus voltage in the phase form is calculated using equation (4.128)

$$\left[\bar{\mathbf{V}}_{\mathbf{5}}^{(\mathbf{abc})}(\mathbf{F})\right] = \begin{bmatrix} 0.0\angle 0^0 \\ 1.087\angle -128.64^0 \\ 1.087\angle -128.64^0 \end{bmatrix} pu$$

Observe that the phase voltage of the faulted phase 'a' is **zero** due to a zero impedance fault.

Line Currents

The sequence components of line currents during fault are calculated using equation (4.129), written here in compact form as

$$\begin{bmatrix} \bar{I}_{ij}^{(0)}(F) \\ \bar{I}_{ij}^{(1)}(F) \\ \bar{I}_{ij}^{(2)}(F) \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{z}_{ij}^{(0)}} & 0 & 0 \\ 0 & \frac{1}{\bar{z}_{ij}^{(1)}} & 0 \\ 0 & 0 & \frac{1}{\bar{z}_{ij}^{(2)}} \end{bmatrix} \begin{bmatrix} \bar{V}_i^{(0)}(F) - \bar{V}_j^{(0)}(F) \\ \bar{V}_i^{(1)}(F) - \bar{V}_j^{(1)}(F) \\ \bar{V}_i^{(2)}(F) - \bar{V}_j^{(2)}(F) \end{bmatrix} \tag{4.132}$$

In equation (4.132), the line is between $\mathbf{i^{th}}$ and $\mathbf{j^{th}}$ buses.

$\bar{z}_{ij}^{(0)}, \bar{z}_{ij}^{(1)}, \bar{z}_{ij}^{(2)}$ represent the respective sequence impedances of the line $\mathbf{i} \longrightarrow \mathbf{j}$

$\bar{V}_i^{(0)}(F), \bar{V}_i^{(1)}(F), \bar{V}_i^{(2)}(F), \bar{V}_j^{(0)}(F), \bar{V}_j^{(1)}(F), \bar{V}_j^{(2)}(F)$ are the sequence components of voltages of $\mathbf{i^{th}}$ and $\mathbf{j^{th}}$ buses respectively during fault.

Line 1: The sequence components of line current are

$$\begin{bmatrix} \bar{I}_{34}^{(0)}(F) \\ \bar{I}_{34}^{(1)}(F) \\ \bar{I}_{34}^{(2)}(F) \end{bmatrix} = \begin{bmatrix} \frac{1.0}{j0.3} & 0 & 0 \\ 0 & \frac{1.0}{j0.10} & 0 \\ 0 & 0 & \frac{1.0}{j0.10} \end{bmatrix} \begin{bmatrix} -0.1538 - (-0.3076) \\ 0.8077 - 0.8077 \\ -0.1923 - (-0.1923) \end{bmatrix}$$

Or,

$$\boxed{\left[\mathbf{\bar{I}_{34}^{(012)}(F)}\right] = \begin{bmatrix} -j0.5128 \\ 0 \\ 0 \end{bmatrix} pu}$$

The line current in phase form is calculated as:

$$\boxed{\left[\mathbf{\bar{I}_{34}^{(abc)}(F)}\right] = \begin{bmatrix} 0.5128\angle -90^0 \\ 0.5128\angle -90^0 \\ 0.5128\angle -90^0 \end{bmatrix} pu}$$

Line 2: The sequence components of line current are

$$\begin{bmatrix} \bar{I}_{35}^{(0)}(F) \\ \bar{I}_{35}^{(1)}(F) \\ \bar{I}_{35}^{(2)}(F) \end{bmatrix} = \begin{bmatrix} \frac{1.0}{j0.3} & 0 & 0 \\ 0 & \frac{1.0}{j0.10} & 0 \\ 0 & 0 & \frac{1.0}{j0.10} \end{bmatrix} \begin{bmatrix} -0.1538 - (-0.4615) \\ 0.8077 - 0.7308 \\ -0.1923 - (-0.2692) \end{bmatrix}$$

Or,

$$\left[\bar{\mathbf{I}}_{\mathbf{35}}^{(\mathbf{012})}(\mathbf{F})\right] = \begin{bmatrix} -j1.0256 \\ -j0.7692 \\ -j0.7692 \end{bmatrix} pu$$

The line current in phase form is calculated as:

$$\left[\bar{\mathbf{I}}_{\mathbf{35}}^{(\mathbf{abc})}(\mathbf{F})\right] = \begin{bmatrix} 2.5641\angle -90^0 \\ 0.2564\angle -90^0 \\ 0.2564\angle -90^0 \end{bmatrix} pu$$

Line 3: The sequence components of line current are

$$\begin{bmatrix} \bar{I}_{45}^{(0)}(F) \\ \bar{I}_{45}^{(1)}(F) \\ \bar{I}_{45}^{(2)}(F) \end{bmatrix} = \begin{bmatrix} \frac{1.0}{j0.3} & 0 & 0 \\ 0 & \frac{1.0}{j0.10} & 0 \\ 0 & 0 & \frac{1.0}{j0.10} \end{bmatrix} \begin{bmatrix} -0.3077-(-0.4615) \\ 0.8077-0.7308 \\ -0.1923-(-0.2692) \end{bmatrix}$$

Or,

$$\left[\bar{\mathbf{I}}_{\mathbf{45}}^{(\mathbf{012})}(\mathbf{F})\right] = \begin{bmatrix} -j0.5128 \\ -j0.7692 \\ -j0.7692 \end{bmatrix} pu$$

The line current in phase form is calculated as:

$$\left[\bar{\mathbf{I}}_{\mathbf{45}}^{(\mathbf{abc})}(\mathbf{F})\right] = \begin{bmatrix} 2.0513\angle -90^0 \\ 0.2564\angle -90^0 \\ 0.2564\angle -90^0 \end{bmatrix} pu$$

Transformer Currents

Transformer $\mathbf{T_1}$: The sequence components of line current are

$$\begin{bmatrix} \bar{I}_{13}^{(0)}(F) \\ \bar{I}_{13}^{(1)}(F) \\ \bar{I}_{13}^{(2)}(F) \end{bmatrix} = \begin{bmatrix} \frac{1.0}{j0.05} & 0 & 0 \\ 0 & \frac{1.0}{j0.05} & 0 \\ 0 & 0 & \frac{1.0}{j0.05} \end{bmatrix} \begin{bmatrix} -0.0769-(-0.1538) \\ 0.8462-0.8077 \\ -0.1538-(-0.1923) \end{bmatrix}$$

Or,

$$\left[\bar{\mathbf{I}}_{\mathbf{13}}^{(\mathbf{012})}(\mathbf{F})\right] = \begin{bmatrix} -j1.538 \\ -j0.7692 \\ -j0.7692 \end{bmatrix} pu$$

The line current in phase form is calculated as:

$$\left[\bar{\mathbf{I}}_{\mathbf{13}}^{(\mathbf{abc})}(\mathbf{F})\right] = \begin{bmatrix} 3.0769\angle -90^0 \\ 0.7692\angle -90^0 \\ 0.7692\angle -90^0 \end{bmatrix} pu$$

Transformer $\mathbf{T_2}$: The sequence components of line current are

$$\begin{bmatrix} \bar{I}_{24}^{(0)}(F) \\ \bar{I}_{24}^{(1)}(F) \\ \bar{I}_{24}^{(2)}(F) \end{bmatrix} = \begin{bmatrix} \frac{1.0}{\infty} & 0 & 0 \\ 0 & \frac{1.0}{j0.05} & 0 \\ 0 & 0 & \frac{1.0}{j0.05} \end{bmatrix} \begin{bmatrix} 0-(-0.3076) \\ 0.8462-0.8077 \\ -0.1538-(-0.1923) \end{bmatrix}$$

$$\left[\bar{\mathbf{I}}_{\mathbf{24}}^{(\mathbf{012})}(\mathbf{F})\right] = \begin{bmatrix} 0 \\ -j0.7692 \\ -j0.7692 \end{bmatrix} pu$$

The line current in phase form is calculated as:

$$\left[\bar{\mathbf{I}}_{\mathbf{24}}^{(\mathbf{abc})}(\mathbf{F})\right] = \begin{bmatrix} 1.538\angle -90^0 \\ 0.7692\angle -90^0 \\ 0.7692\angle -90^0 \end{bmatrix} pu$$

Generator Currents

The sequence components of generator currents during fault are calculated using the expression

$$\begin{bmatrix} \bar{I}_{Gi}^{(0)}(F) \\ \bar{I}_{Gi}^{(1)}(F) \\ \bar{I}_{Gi}^{(2)}(F) \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{z}_{gi}^{(0)}} & 0 & 0 \\ 0 & \frac{1}{\bar{z}_{gi}^{(1)}} & 0 \\ 0 & 0 & \frac{1}{\bar{z}_{gi}^{(2)}} \end{bmatrix} \begin{bmatrix} \bar{E}_{Gi}^{(0)}(F) - \bar{V}_{ti}^{(0)}(F) \\ \bar{E}_{Gi}^{(1)}(F) - \bar{V}_{ti}^{(1)}(F) \\ \bar{E}_{Gi}^{(2)}(F) - \bar{V}_{ti}^{(2)}(F) \end{bmatrix} \tag{4.133}$$

where,

$\bar{E}_{Gi}^{(0)}(F), \bar{E}_{Gi}^{(1)}(F), \bar{E}_{Gi}^{(2)}(F)$ the zero, positive and negative sequence generated voltages respectively of $\mathbf{i^{th}}$ generator.

$\bar{V}_{ti}^{(0)}(F), \bar{V}_{ti}^{(1)}(F), \bar{V}_{ti}^{(2)}(F)$ are the zero, positive and negative sequence terminal voltages respectively of $\mathbf{i^{th}}$ generator after fault.

$\bar{z}_{gi}^{(0)}(F), \bar{z}_{gi}^{(1)}(F)$ and $\bar{z}_{gi}^{(2)}(F)$ are the sequence impedances of the $\mathbf{i^{th}}$ generator.

Generator 1 : The sequence components of generator 1 current are

$$\begin{bmatrix} \bar{I}_{G_1}^{(0)}(F) \\ \bar{I}_{G_1}^{(1)}(F) \\ \bar{I}_{G_1}^{(2)}(F) \end{bmatrix} = \begin{bmatrix} \frac{1.0}{j0.05} & 0 & 0 \\ 0 & \frac{1.0}{j0.20} & 0 \\ 0 & 0 & \frac{1.0}{j0.20} \end{bmatrix} \begin{bmatrix} 0 - (-0.0769) \\ 1 - 0.8462 \\ 0 - (-0.1538) \end{bmatrix}$$

$$\left[\mathbf{\bar{I}_{G_1}^{(012)}(F)}\right] = \begin{bmatrix} -j1.538 \\ -j0.7692 \\ -j0.7692 \end{bmatrix} pu$$

The phase components generator current are calculated as:

$$\left[\mathbf{\bar{I}_{G_1}^{(abc)}(F)}\right] = \begin{bmatrix} 3.0769\angle -90^0 \\ 0.7692\angle -90^0 \\ 0.7692\angle -90^0 \end{bmatrix} pu$$

Generator 2 : The sequence components of Generator 1 current are

$$\begin{bmatrix} \bar{I}_{G_2}^{(0)}(F) \\ \bar{I}_{G_2}^{(1)}(F) \\ \bar{I}_{G_2}^{(2)}(F) \end{bmatrix} = \begin{bmatrix} \frac{1.0}{j0.05} & 0 & 0 \\ 0 & \frac{1.0}{j0.20} & 0 \\ 0 & 0 & \frac{1.0}{j0.20} \end{bmatrix} \begin{bmatrix} 0 - 0 \\ 1 - 0.8462 \\ 0 - (-0.1538) \end{bmatrix}$$

$$\left[\mathbf{\bar{I}_{G_2}^{(012)}(F)}\right] = \begin{bmatrix} 0 \\ -j0.7692 \\ -j0.7692 \end{bmatrix} pu$$

The phase components generator current are calculated as:

$$\left[\mathbf{\bar{I}_{G_2}^{(abc)}(F)}\right] = \begin{bmatrix} 1.538\angle -90^0 \\ 0.7692\angle -90^0 \\ 0.7692\angle -90^0 \end{bmatrix} pu$$

The flow of sequence currents in the sequence networks is shown next in the Fig. 4.72. From Fig. 4.72 the following points are worth observing:

- Both generators contribute equal amount of positive and negative sequence currents as the network is symmetrical as seen from the fault point.
- Since the positive and negative sequence fault voltages are equal for buses 3 and 4, the positive and negative sequence currents through line L_1 between buses 3 and 4 are zero.

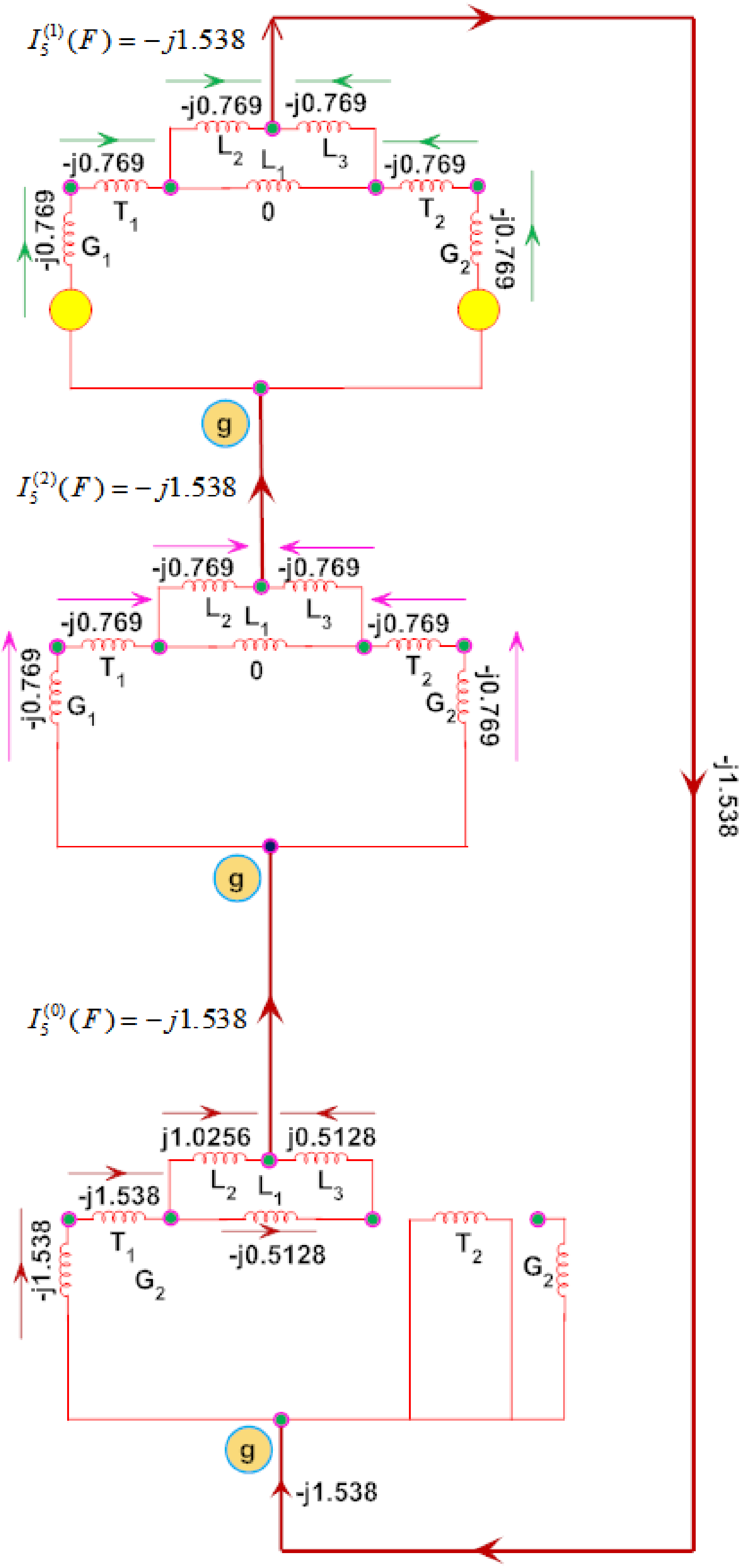

Figure 4.72: Flow of sequence currents for LG fault at bus 5

- The zero sequence circuit of generator G_2 is open circuited due to $\Delta - \Delta$ transformer T_2 as a result, G_2 does not contribute any zero sequence current to the fault. Generator G_1 has to provide the entire zero sequence current.

- Moreover, the zero sequence network is not symmetrical, hence, zero sequence voltages of buses 3 and 4 are not equal and as a result a zero sequence current flows through line L_1.

In the next lecture, we will look into the examples of short circuit fault calculation for LL and LLG faults.